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OBSERVATIONS

Analysis of the List-Strength Effect

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In several articles, Shiffrin et al. (e.g., Shiffrin, Ratcliff, & Clark, 1990) argued that their data on the list-strength effect (LSE), in conjunction with their data on the list-length effect (LLE), are counter to current global matching memory models (GMMs). This is only true if one assumes that the memory system is reinitialized after every list, which is an unrealistic default assumption present in many implementations of GMMs. By making the more reasonable assumption that memory is continuous, it is shown that TODAM (and probably other GMMs) does in fact predict the LSE and LLE data.

The *list-strength effect* (LSE) refers to the effect other items in a list have on the retention of any given item in the list. These other items function as competition for the given item, so the stronger they are, the poorer the retention should be for the given item. As studied by Ratcliff, Clark, and Shiffrin (1990) and Shiffrin, Ratcliff, and Clark (1990), the LSE can be tested by selected comparisons between pure and mixed lists. Pure lists contain only strong or weak items, whereas mixed lists contain both. Strong items in a mixed list (MS) should be retained better than strong items in a pure list (PS) because the competition is less, but weak items in a pure list (PW) should be retained better than weak items in a mixed list (MW) for the same reason. In a series of experiments, Ratcliff et al. (1990) and Murnane and Shiffrin (1991a, 1991b) failed to find an LSE (but see Murnane & Shiffrin, 1991a, Experiment 1). Basically, the result seems to be that the composition of the list does not matter; strong items are better retained than weak items, but equally so in pure and mixed lists.

Because from the competition argument d′(MS) > d′(PS) and d′(MW) < d′(PW), it follows that

$$\frac{d'(MS)}{d'(PS)} > 1.0 \quad \text{and} \quad \frac{d'(MW)}{d'(PW)} < 1.0.$$

Because the first ratio is greater than 1.0 and the second ratio is less than 1.0, it follows that the ratio of the ratios (ROR) should also be greater than 1.0. That is,

$$\text{ROR} = \frac{d'(MS)/d'(PS)}{d'(MW)/d'(PW)} = \frac{d'(MS)}{d'(MW)}\frac{d'(PW)}{d'(PS)} > 1.0.$$

Because $d' = \mu_O/\sigma_N$, one can write

$$\text{ROR} = \frac{\mu(MS)}{\mu(PS)}\frac{\mu(PW)}{\mu(MW)}\frac{\sigma(MW)}{\sigma(MS)}\frac{\sigma(PS)}{\sigma(PW)}. \quad (1)$$

According to global matching memory models (the search of associative memory [SAM] model for recalling of Gillund & Shiffrin, 1984; the MINERVA 2 model of Hintzman, 1986, 1988; the composite holographic associative recall model [CHARM] of Eich, 1982; the theory of distributed associative memory [TODAM] model of Murdock, 1982; and the matrix model of Humphreys, Pike, Bain, & Tehan, 1989), strength is a property of the items themselves independent of the strength of other items in the list. If list length or retention interval is controlled (as it usually is) and there is no rehearsal borrowing, then $\mu(MS) = \mu(PS)$ and $\mu(PW) = \mu(MW)$. Thus, one can simplify ROR to

$$\text{ROR} = \frac{\sigma(MW)}{\sigma(MS)}\frac{\sigma(PS)}{\sigma(PW)}.$$

According to GMMs, all items enter into the comparison process so $\sigma(MW) = \sigma(MS)$. Consequently, the prediction is

$$\text{ROR} = \frac{\sigma(PS)}{\sigma(PW)} > 1.0. \quad (1a)$$

However, in many experiments (e.g., Murnane & Shiffrin, 1991a, 1991b; Ratcliff et al., 1990), the obtained ROR is generally about 1.0, and this would seem to be counter to all GMMs.

Rehearsal borrowing is a possible complication. In a mixed list, rehearsal time may be borrowed from strong items and given to weak items; if this were the case, then $\mu(MW) > \mu(PW)$ and $\mu(MS) < \mu(PS)$. This difference in means could offset the difference in variances and so lead to the obtained results (i.e., ROR values of about 1.0).

However, as argued in Murnane and Shiffrin (1991b), if there is rehearsal borrowing from strong to weak on a mixed list, then on a final recognition test in which list identity is lost so $\sigma(MW) \cong \sigma(PW)$ and $\sigma(MS) \cong \sigma(PS)$, then a negative LSE (R < 1.0) would be predicted; Equation 1 simplifies to

$$\text{ROR} = \frac{\mu(MS)}{\mu(PS)}\frac{\mu(PW)}{\mu(MW)} < 1.0.$$

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In four experiments, Murnane and Shiffrin failed to find a negative LSE in final recognition, so they concluded that rehearsal borrowing was not counteracting a genuine LSE on the immediate recognition test.

Yonelinas, Hockley, and Murdock (1992) used a more direct approach to the study of rehearsal borrowing. They used rapid sequential visual presentation in which items were presented at rates up to 10 items per second, and the hope was that no rehearsal borrowing would be possible. In fact, in the mixed list, there may have been reverse borrowing (rehearsal or attention redirected from the weak to the strong) leading (in two experiments) to ROR values greater than 1.0. However, when reverse borrowing was made more difficult (by a blocked design), and in other experiments using a forced-choice rather than a yes–no procedure, the ROR values did not differ significantly from 1.0.

Consequently, the whole issue seems to come down to the pure-strong to pure-weak variance ratio. If this ratio is greater than 1.0, then you should get an LSE; but if this ratio were 1.0, then the ROR should also be 1.0. We address this issue in this article.

TODAM

The whole thrust of the LSE enterprise is to test GMMs on a common ground; namely, recognition memory (Ratcliff, Sheu, & Gronlund, 1992). Consequently, even though many of the LSE experiments used a paired-associate format, we shall restrict our TODAM analysis to the item-recognition case. The additional complications introduced by the paired-associate format are beyond the scope of this article.

In TODAM, all items are stored in a common memory vector **M**. If **M** is initialized to zero at the start of each list, then it will in fact only be the items in the current list that enter into the comparison process so $\sigma(\mathrm{PS}) > \sigma(\mathrm{PW})$ and by Equation 1a ROR > 1.0. This is the default assumption used in many implementations of TODAM. But **M** cannot be initialized to zero at the start of each list; if it were, performance on a final recognition test would be at chance. Not only that, but subjects would not even be able to remember the task instructions, that they were in an experiment, or what their own name was. This seems rather unlikely.

Instead, it seems more reasonable to assume that memory is continuous. The memory vector is not initialized to zero at the start of each list or even at the start of the experiment. (For evidence, see Estes, 1991.) Instead, it is continuous from the past to the present, and the fact that there is a definite starting time for the experimental situation does not alter this. The information entered in memory is different, and subjects can discriminate preexperimental from experimental events, but both types of information are contained in the common memory vector.

Can TODAM then predict the results from the LSE experiments? The problem is to show not only why one does not get an LSE, but also why one does get an LLE in comparable experiments or conditions. That is, performance as measured by d′ does get worse as list length increases. It turns out that when one works out the derivations of the model with the continuous memory assumption and uses reasonable parameter values, this is exactly the pattern of data one would predict. No new additions or modifications to the model are needed. What one does need, however, is the continuous memory assumption, and this is critical.

We use the standard TODAM model for item recognition (e.g., Murdock & Lamon, 1988). Items are represented as random vectors in an N-dimensional space and stored in a common memory vector **M**. Features of the item are encoded probabilistically so any given feature is encoded with probability p or not encoded (set to zero) with probability $1 - p$. As every new item is added to the memory vector, the memory vector is decremented by alpha where alpha is the forgetting parameter. If $\mathbf{f}_j$ is the jth item in a list of L items, then the storage equation is

$$\mathbf{M}_j = \alpha \mathbf{M}_{j-1} + p\mathbf{f}_j.$$

The encoding probability p varies with presentation duration increasing from zero at some very short duration to 1 at some very long duration. However, it does not vary from item to item or from trial to trial when items are repeated. The forgetting parameter alpha does not vary with presentation duration or from trial to trial; it is the same for novel items and repeated items.

A probe item is compared with the memory vector by the dot (or inner) product where for item **f** and the memory vector **M** the dot product is defined as

$$\mathbf{f} \cdot \mathbf{M} = \sum_{i=1}^{N} f(i)M(i).$$

The dot product assesses the strength of the item in memory or gives the familiarity of the probe item and so serves as the basic information for a decision in a recognition test. Because **M** is a common memory vector, after a single presentation of a list of L items

$$\mathbf{M}_L = \sum_{l=1}^{L} p\mathbf{f}_l$$

so all the items enter into the comparison process. This is what makes TODAM a GMM (Humphreys et al., 1989).

As is customary, d′ is defined as the ratio of the old-item mean to the new-item standard deviation σ_N. If one neglects output interference (the interference or variance from prior test items), then for a list of L items presented R times

$$\sigma_N^2 = \frac{1}{N}\frac{1-\alpha^{2L}}{1-\alpha^2} \cdot \left[p\frac{1-\alpha^{2RL}}{1-\alpha^{2L}} + p^2\left\{\left(\frac{1-\alpha^{RL}}{1-\alpha^{L}}\right)^2 - \frac{1-\alpha^{2RL}}{1-\alpha^{2L}}\right\}\right]$$

(Murdock, 1992; Murdock & Lamon, 1988). If R = 1, this simplifies to

$$\sigma_N^2 = \frac{p}{N}\frac{1-\alpha^{2L}}{1-\alpha^2}.$$

For the old-item mean, from the basic storage equation $\mu_k = p\alpha^{L-k}$ so

$$\mu_O = \frac{1}{L}\sum_{l=1}^{L}\mu_l = \frac{p}{L}\frac{1-\alpha^L}{1-\alpha}.$$

Thus, given the parameters of the model (N, p, and α) and the experimental conditions (R and L), one can compute an expected d′ value that one can compare with the d′ value obtained from an experiment.

This assumes memory is not continuous; the memory vector is set to zero at the start of each list. If instead one assumes memory is continuous, if H "items" are presented before the start of the experimental session, the experiment has S lists of length L, q is the encoding probability for the preexperimental items, and p is the encoding probability for the experimental items, then after S experimental lists have been presented

$$\sigma^2(H, S, L) = \frac{1}{N}\left\{q\alpha^{2SL}\frac{1-\alpha^{2H}}{1-\alpha^2} + p\frac{1-\alpha^{2SL}}{1-\alpha^2}\right\}.$$

That is, the first term $[q(1-\alpha^{2H})/(1-\alpha^2)]$ is the variance of the preexperimental items, and the second term $[p(1-\alpha^{2SL})/(1-\alpha^2)]$ is the variance of the experimental items. With independent items, the variance of a sum is the sum of the variances, but the variance of the preexperimental items is decremented by α^{2SL} resulting from the presentation of S lists of length L.

As H goes to infinity, α^{2H} goes to zero so

$$\sigma_N^2 = \lim_{H\to\infty}\sigma^2(H, S, L) = \frac{1}{N(1-\alpha^2)}\{p + \alpha^{2SL}(q-p)\}.$$

Consequently, one can get an explicit expression for d′ for a recognition-memory experiment using a study-test procedure. With the continuous memory assumption, it is

$$d' = \frac{\mu_O}{\sigma_N} = \frac{(p/L)[(1-\alpha^L)/(1-\alpha)]}{\sqrt{[1/N(1-\alpha^2)]\{p + \alpha^{2SL}(q-p)\}}}. \quad (2)$$

By Equation 1a, if list length is controlled, then the ROR is

$$ROR = \sigma(PS)/\sigma(PW).$$

Neglecting output interference, when strength is varied by presentation duration in a mixed-list design, assuming the usual counterbalancing conditions, if p_1 and p_2 are the strong (slow) and weak (fast) encoding probabilities and p^* is the encoding probability for the Sth list, then

$$\sigma_N^2(\text{pure}) = \frac{1}{N(1-\alpha^2)} \cdot \left[q\alpha^{2SL} + \frac{p_1+p_2}{2}\{(1-\alpha^{2SL}) - (1-\alpha^{2L})\} + p^*(1-\alpha^{2L})\right].$$

The term in braces simply subtracts the current list from all the experimental lists to date, and this is weighted by the average of p_1 and p_2. The last term is the current list weighted by p^*. Generally, p^* would be p_1 for a strong list, but p_2 for a weak list.

All one has to do then is to form the ratio for ROR and, by Equation 1a, if p_1 is strong and p_2 is weak, then one has

$$ROR = \sqrt{\frac{q\alpha^{2SL} + [(p_1+p_2)/2](\alpha^{2L}-\alpha^{2SL}) + p_1(1-\alpha^{2L})}{q\alpha^{2SL} + [(p_1+p_2)/2](\alpha^{2L}-\alpha^{2SL}) + p_2(1-\alpha^{2L})}}. \quad (3)$$

Values of ROR as a function of S are shown in Table 1 for $\alpha = 0.995$, L = 15, q = 0.7, $p_1 = 0.5$, and $p_2 = 0.3$. As can be seen, the values of ROR are close to 1.0. In addition, there is little build-up of proactive inhibition as the ROR does not increase much over trials.

It is not hard to understand why the value of ROR stays so close to 1.0. For any value of S, both the numerator and the denominator are a linear combination of three factors: the preexperimental variance; the variance from all prior experimental lists, strong and weak; and the current list. Only the current list is greater for a slow presentation than for a fast presentation, but it is dwarfed by the two other components.

Why did we choose a value of alpha so close to 1.0? This is exactly the value of alpha we needed for an application of the model to some paired-associate data (Murdock & Hockley, 1989), and because TODAM stores item and associative information in a common memory vector, it seems reasonable to assume the same alpha value applies here too. Why is q greater than p_1? Even strong items in an experimental situation are probably weaker than average items in everyday situations, and this is why we make $q > p_1$.

Actually, for the LSE, this implementation of the continuous memory assumption is conservative. We are differentially weighting the prior and the experimental variances by q and p, and the larger the prior-item variance the less impact any experimental manipulation will have. Prior experimental items will surely be repeated so the repetition covariance component inflates the prior noise level considerably. By neglecting this covariance component, we are in effect magnifying the contribution of the experimental component, and still the value of ROR is insignificantly different from 1.0.

Table 1
Predicted Values of the Ratio of the Ratios (ROR) From Equation 3 as a Function of S for $\alpha = 0.995$, L = 15, q = 0.7, $p_1 = 0.5$, and $p_2 = 0.3$

S	ROR
1	1.021
4	1.025
7	1.028
10	1.030
13	1.032
.	
.	
.	
100	1.036

Suppose strength is varied by repetition? Then the derivation is somewhat more complex; thus, it is presented in three Appendixes. Appendix A presents a general linearity principle that is useful in working out derivations in such cases. The variance of a sum of conditions is the sum of the variances of the separate conditions, each weighted by an interference coefficient in which the interference coefficients reflect the lag of each condition. Appendix B derives an explicit expression for the variance of a mixed list, and Appendix C derives explicit expressions for the pure-strong, pure-weak, and mixed lists averaged over counterbalancing conditions. Table 2 shows the predicted ROR values with the same parameter values, and again the same results obtain. The predicted values of ROR are close to 1.0, and they do not increase very much over blocks (B) of three trials.

The predicted value of ROR does increase slightly as list length or number of repetitions increases. For instance, with five presentations and a 20-item list, with these same parameters in the limit (i.e., as $B \rightarrow \infty$) the predicted value of ROR is 1.205. In Experiment 2, using a list length of 20 and five presentations, Ratcliff et al. (1992) reported an ROR value for their group data of 1.21 and 1.13 for the average of the individual S's RORs, so the prediction seems quite close.

What about the LLE? We would argue that one reason the LLE comes about is because, on average, the study–test lag is greater in long lists than in short lists so the mean is different, but by the continuous memory assumption the variances are essentially the same. If we neglect output interference, to assess the magnitude of the LLE for any given set of parameter values, we would want d′(k) where k is the input serial position. Because d′ is the old-item mean over the new-item standard deviation, for d′(k), we would have

$$d'(k) = \frac{p\alpha^{L-k}}{\sqrt{[p + \alpha^{2SL}(q - p)]/N(1 - \alpha^2)}}. \qquad (4)$$

Predicted values for d′(k) are shown in Table 3, so obviously one gets an LLE using the same parameter values that generate a negligible LSE.

Do these parameter values give reasonable estimates of d′ for study–test data? Study–test d′ values from Equation 2, which use these same parameter values with L = 20 and N = 1,000 are shown in Table 4. These are certainly good ballpark estimates for study–test data when one varies presentation duration (e.g., Murdock & Anderson, 1975), so again the predictions seem quite reasonable.[1]

Table 2
Predicted Values of the Ratio of the Ratios (ROR) as a Function of Blocks (B) of Three Trials When Strength Is Varied by Repetition (From Appendix 3 With Parameter Values $\alpha = 0.995$, $q = 0.7$, $p = 0.4$, $H = 1{,}000$, $L = 15$, and $R = 3$)

B	ROR
1	1.026
2	1.063
3	1.080
4	1.087
.	
.	
.	
100	1.092

Table 3
The List-Length Effect From Equation 4 for $L = 40$, $q = 0.7$, $p = 0.4$, $S = 15$, $\alpha = 0.995$, and $N = 1{,}000$

k	d′
40	2.00
30	1.90
20	1.81
10	1.72
1	1.64

Note. Shown is d′(k), where k is the input serial position.

If memory is continuous, and the value of alpha is so high, how is list discrimination possible? List discrimination would be a joint function of the separation between the two lists and the age of the more recent list. Consequently, list discrimination on the basis of the d′ values for the two items in a two-alternative, forced-choice judgment of recency would increase with the list separation but decrease with the age of the more recent list. The d′ values would depend on the exact experimental conditions, but if the predicted judgment-of-recency values were too low, it could be that a value of N of 1,000 is too low. A larger value of N would facilitate list discrimination without altering the LSE–LLE pattern in any way. However, before working out quantitative details for judgment-of-recency data, we need to include output interference in the model because output effects are, if anything, larger than input effects in the data (Murdock & Anderson, 1975).

What happens to the LLE in a final recognition test? With the continuous memory assumption, the only difference would be in the lag of an in-session or end-session test so the LLE should be attenuated if not eliminated in an end-session test. That is, an LLE can be generated by lag differences, and if x and y are the lag differences for short and long lists, $\alpha^y < \alpha^x$, if $x < y$. For an end-session test, the ratio of x to y will be much less than for an in-session test so the LLE will be attenuated. However, one needs to represent output interference in the model before a quantitative comparison is possible.

Does the LLE disappear in the final recognition test? The hit rate (HR) data for short and long lists for in- and end-sessions tests from Murnane and Shiffrin (1991b) are shown in Table 5. One cannot directly compare in- and end-session d′ values for short and long lists because one has separate false-alarm rates (FAR) for short and long lists tested in-session, but a common FAR for short and long lists tested end-session. It may instead be that the end-session LLE is

[1] For the simulation, the value of N was 1,000, which may seem large compared with some previous applications. However, for paired-associate learning data, we needed an N of 5,000 so a value of N of 1,000 is conservative.

Table 4
Study–Test d′ Values From Equation 2 as a Function of p for α = 0.995, L = 20, S = 15, q = 0.7, and N = 1,000

p	d′
0.5	2.11
0.3	1.60

attenuated but not eliminated; it is hard to say because of the obvious floor effect (chance HR is .50).[2]

It could be argued that by using the continuous memory assumption, TODAM is now a different model, and there is no guarantee that this new model explains any of the effects handled by the previous model. We would disagree with both of these points. First, we have only changed the assumption about the starting value of the storage equation; the storage equation itself remains unchanged, the basic encoding process (probabilistic encoding of random vectors) is unchanged, the retrieval operation (the dot product) is unchanged, the parameters are unchanged, and even the parameter values are essentially the same as in previous applications.

The storage equation is a difference equation, and any difference (or differential) equation has starting values and boundary conditions. All we have done is to change the default assumption, emphasize that the starting value is not—in fact cannot be—zero, and work out the implications. Moreover, even this is not a new development. In the TODAM simulations reported in Murdock and Lamon (1988), the memory vector was filled with random noise before presentation of the first list and carried over from trial to trial.

Second, there is no reason to worry that this change would affect previous applications. This change only affects the variance, not the means, and most if not all of our previous applications relied on mean values. If one uses the continuous memory assumption, then the variance will be larger, but this could be counteracted by increasing N, and increases in N have no effect whatsoever on mean values. In fact, the increase in N could offset any variance changes due to the continuous memory assumption, so we feel concerns about a different model or different predictions are not serious matters.

Summary

Using the standard TODAM model for item recognition, we have derived explicit expressions for the LSE statistics when strength is manipulated by stimulus duration or repetition. With the continuous memory assumption, TODAM not only can predict the general pattern of data in studies of the LSE, but also gives reasonable quantitative estimates for the LSE, the LLE, and the d′ values for a study–test procedure with the same parameter values. Furthermore, the predictions are much the same regardless of whether strength is varied by presentation duration or by repetition.

In the various articles by Shiffrin and his colleagues (e.g., Shiffrin et al., 1990), they made some general statements about the inability of the GMMs to predict the LSE, but they did not work out the detailed predictions. For TODAM, their statements only apply when memory is assumed to be initialized to zero at the start of every list. As we argued here, this assumption is unrealistic. By using the more reasonable continuous memory assumption, one sees that TODAM (and probably other GMMs too) can in fact predict the results quite accurately.

Table 5
Hit Rates for Short and Long Lists Tested In- or End-Session for the Four Experiments of Murnane and Shiffrin (1991b)

	In-session		End-session	
Experiment	Short	Long	Short	Long
1	.77	.69	.55	.52
2	.795	.69	.565	.545
3	.685	.705	.54	.54
4	.76	.69	.56	.54

[2] Why does one get an LSE in recall? In free recall, there is output interference so in a mixed list if strong items are recalled before weak items, then the weak items will suffer. With cued recall, there is little or no output interference so one probably should not expect an LSE in cued recall. Ratcliff, Clark, and Shiffrin (1990) found no LSE in either recognition or cued recall in Experiment 3, but a small LSE (ROR = 1.38) in cued recall. However, Metcalfe (personal communication, April 3, 1989) did some simulations with CHARM in which she found an LSE in cued recall but not in item recognition.

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Appendix A

In this Appendix, we derive a general principle for computing the variance of a multicondition experiment using a within-lists design. If the variance of a set of items (or lists) is σ^2, then the variance of this same set of items followed by another set of (independent) items is

$$\sigma_A^2 = c^2\sigma^2$$

where $\sigma_A{}^2$ is the attenuated variance and c depends on the number of items (or lists) in the interfering set.

Say there are L items in the first (target) set and the storage equation is

$$\mathbf{M}_j = \alpha\mathbf{M}_{j-1} + \mathbf{f}_j.$$

Then the familiarity or strength s of any probe item is the dot product of the probe item with the memory vector **M**. For a new-item probe **f** (no subscript),

$$S = \mathbf{f} \cdot \mathbf{M}.$$

Because the items are represented by random vectors, this strength has a mean and a variance; namely, E[s] and Var[s] where

$$s = \mathbf{f} \cdot \mathbf{M} \qquad \text{and} \qquad \mathbf{M} = \sum_{l=1}^{L} c_l\mathbf{f}_l$$

where c_l is the serial-position constant of the lth item. Consequently,

$$E[s] = E[\mathbf{f} \cdot \mathbf{M}] = E\left[\mathbf{f} \cdot \sum_{l=1}^{L} c_l\mathbf{f}_l\right].$$

and

$$\mathrm{Var}[s] = \mathrm{Var}[\mathbf{f} \cdot \mathbf{m}] = \mathrm{Var}\left[\mathbf{f} \cdot \sum_{l=1}^{L} c_l\mathbf{f}_l\right].$$

Now if these items are followed by T other items, then

$$E[s] = E[\mathbf{f} \cdot \mathbf{M}] = E\left[\mathbf{f} \cdot \left(\sum_{l=1}^{L} c_l\mathbf{f}_l + \sum_{t=1}^{T} c_t\mathbf{f}_t\right)\right]$$

$$= E\left[\mathbf{f} \cdot \sum_{l=1}^{L} c_l\mathbf{f}_l\right] + E\left[\mathbf{f} \cdot \sum_{t=1}^{T} c_t\mathbf{f}_t\right]$$

and

$$\mathrm{Var}[s] = \mathrm{Var}[\mathbf{f} \cdot \mathbf{M}] = \mathrm{Var}\left[\mathbf{f} \cdot \left\{\sum_{l=1}^{L} c_l\mathbf{f}_l + \sum_{t=1}^{T} c_t\mathbf{f}_t\right\}\right]$$

$$= \mathrm{Var}\left[\mathbf{f} \cdot \sum_{l=1}^{L} c_l\mathbf{f}_l\right] + \mathrm{Var}\left[\mathbf{f} \cdot \sum_{t=1}^{T} c_t\mathbf{f}_t\right]$$

because there is no covariance (the items are assumed to be independent).

Compare the variance of the L items alone with the variance of the L items followed by T additional items. Let the variance of the alone condition be σ^2; then

$$\sigma^2 = \mathrm{Var}\left[\mathbf{f} \cdot \sum_{l=1}^{L} c_l\mathbf{f}_l\right] = \sum_{l=1}^{L} c_l^2\mathrm{Var}[\mathbf{f} \cdot \mathbf{g}] = N\sum_{l=1}^{L} c_l^2\sigma^4 = \frac{1}{N}\sum_{l=1}^{L} c_l^2.$$

Let the variance of these same items in the interference condition be σ_I^2; then

$$\sigma_I^2 = \mathrm{Var}\left[\mathbf{f} \cdot \sum_{l=1}^{L} d_l\mathbf{f}_l\right]$$

where $d_l = \alpha^T c_l$. (The expression for d_l is a direct consequence of the storage equation.) Consequently,

$$\sigma_I^2 = \mathrm{Var}\left[\mathbf{f} \cdot \sum_{l=1}^{L} d_l\mathbf{f}_l\right] = \mathrm{Var}\left[\mathbf{f} \cdot \sum_{l=1}^{L} \alpha^T c_l\mathbf{f}_l\right]$$

$$= \alpha^{2T}\sum_{l=1}^{L} c_l^2\mathrm{Var}[\mathbf{f} \cdot \mathbf{g}] = \alpha^{2T}\frac{1}{N}\sum_{l=1}^{L} c_l^2$$

so

$$\sigma_I^2 = \alpha^{2T}\sigma^2.$$

Thus, in the interference condition, the alone variance is reduced by a factor of α^{2T}.

More generally, the same argument applies whether there are lists of items rather than single items or whether there are several sets of items or lists, not just two. If there is repetition within one set, then there will be covariance within that set but not across sets. As

an example, if $\mathbf{g}$ is one set and $\mathbf{h}_1$ and $\mathbf{h}_2$ are the two presentations within the second set, then

$$\begin{aligned}\mathrm{Var}[\mathbf{f}\cdot(\mathbf{g}+\mathbf{h}_1+\mathbf{h}_2)] &= \mathrm{Var}[\mathbf{f}\cdot\mathbf{g}+\mathbf{f}\cdot\mathbf{h}_1+\mathbf{f}\cdot\mathbf{h}_2]\\ &= \mathrm{Var}[\mathbf{f}\cdot\mathbf{g}]+\mathrm{Var}[\mathbf{f}\cdot\mathbf{h}_1]+\mathrm{Var}[\mathbf{f}\cdot\mathbf{h}_2]+2\,\mathrm{Cov}[\mathbf{f}\cdot\mathbf{h}_1,\mathbf{f}\cdot\mathbf{h}_2],\end{aligned}$$

but there is neither $\mathbf{f}\cdot\mathbf{g}$, $\mathbf{f}\cdot\mathbf{h}_1$ nor $\mathbf{f}\cdot\mathbf{g}$, $\mathbf{f}\cdot\mathbf{h}_2$ covariance.

The implication of this analysis for the repetition manipulation of the LSE design is that if one has explicit expressions for the variance of the three conditions separately, then one can obtain an explicit expression for the new-item variance at any point in the experimental session by summing all occurrences to date of these three conditions, each weighted by an interference coefficient that depends only on the total number of subsequent lists for that condition, as these subsequent lists are the interference for that condition. To handle counterbalancing in the order of list presentation, one must average these coefficients over all possible permutations.

Appendix B

In this Appendix, we derive an explicit expression for the mixed-list condition of the LSE design when strength is varied by repetition. If $\mathbf{f}$ is an N-dimensional item vector, then the standard TODAM storage equation would be

$$\mathbf{M}_j = \alpha\mathbf{M}_{j-1} + p\mathbf{f}_j$$

where $\mathbf{M}$ is the memory vector, α is the forgetting parameter, and p is the encoding probability. Each element in a given item vector is encoded (added to the memory vector) with probability p or not encoded (not added to the memory vector) with probability $1-p$. See Murdock and Lamon (1988) for details.

For a single presentation of a list of L items (W condition), the total strength of all items is simply

$$\sum_{l=1}^{L} p\alpha^{L-l} = p\frac{1-\alpha^L}{1-\alpha}.$$

Over counterbalancing or randomization conditions, all items should have the same average strength so $\mu(\mathrm{W})$, the mean item strength for the W condition, should be

$$\mu(\mathrm{W}) = \frac{1}{L}\sum_{l=1}^{L} p\alpha^{L-l} = \frac{p}{L}\frac{1-\alpha^L}{1-\alpha}.$$

For R presentations of a list of L items (S condition), the total strength of all items is

$$\sum_{r=1}^{R}\sum_{l=1}^{L} p\alpha^{(R-r)L+L-l} = p\frac{1-\alpha^{RL}}{1-\alpha}.$$

Again, over replications, all items should have the same average strength so

$$\mu(\mathrm{S}) = \frac{1}{L}\sum_{r=1}^{R}\sum_{l=1}^{L} p\alpha^{(R-r)L+L-l} = \frac{p}{L}\frac{1-\alpha^{RL}}{1-\alpha}.$$

For the mixed (M) condition, assume m items are presented R times and m items are presented once for a total of T presentations, where $T = mR + m = (R+1)m$. As above, the total strength will be

$$\sum_{t=1}^{T} p\alpha^{T-t} = p\frac{1-\alpha^T}{1-\alpha}.$$

Any given once-presented (weak) item is equally likely to be presented in the T possible serial positions so

$$\mu(\mathrm{MW}) = \frac{p}{T}\frac{1-\alpha^T}{1-\alpha}.$$

The remaining strength values must be equally divided among the m repeated items so

$$\mu(\mathrm{MS}) = \frac{p}{m}\left(\frac{1-\alpha^T}{1-\alpha} - \frac{m}{T}\frac{1-\alpha^T}{1-\alpha}\right) = \frac{p(T-m)}{mT}\frac{1-\alpha^T}{1-\alpha}.$$

These derivations for $\mu(\mathrm{W})$, $\mu(\mathrm{S})$, $\mu(\mathrm{MW})$, and $\mu(\mathrm{MS})$ apply to in-session tests, and they assume there is no output (test) interference. They apply regardless of whether one assumes the memory vector is cumulative over lists.[3]

For the variance, if one assumes that the memory vector is cleared after each list, then for the W condition

$$\sigma^2(\mathrm{W}) = \frac{1}{N}p\frac{1-\alpha^{2L}}{1-\alpha^2};$$

and for the S condition, if items are presented in a fixed presentation order, then

$$\sigma^2(\mathrm{S}) = \frac{1}{N}\frac{1-\alpha^{2L}}{1-\alpha^2}\cdot\left[p\frac{1-\alpha^{2RL}}{1-\alpha^{2L}} + p^2\left\{\left(\frac{1-\alpha^{RL}}{1-\alpha^L}\right)^2 - \frac{1-\alpha^{2RL}}{1-\alpha^{2L}}\right\}\right]$$

(see Murdock, 1992; Murdock & Lamon, 1988, for details). Computer simulation suggests that the same expression for $\sigma^2(\mathrm{S})$ characterizes a randomized presentation order, at least if a trials format is preserved.

The variance of a sum of random variables is the sum of the variances plus twice the covariance. With independent item vectors, the covariance comes in when items are repeated across trials and, with probabilistic encoding, this is proportional to p^2 not p. So, the expression for σ^2 is the exact counterpart of the form

[3] The notion of total item strength provides a way of simplifying the TODAM derivations. What we really have is

$$\mu_k = \mathrm{E}[\mathbf{f}_k\cdot\mathbf{M}] \quad\text{and}\quad \mu_O = \frac{1}{L}\sum_{l=1}^{L}\mu_l$$

so the average old-item strength is simply the average of an old-item probe $\mathbf{f}_k$ dotted with the memory vector $\mathbf{M}$ (see e.g., Murdock, 1992; Murdock & Lamon, 1988, for a more complete presentation).

(*Appendixes continue on next page*)

$$\frac{1}{N}\sum_{l=1}^{L} a_l^2\left\{p\sum b_r^2 + p^2\sum_{r\neq r'} b_r b_{r'}\right\}$$

used in Appendix C of Murdock and Lamon (1988).

We can use this to derive an expression for $\sigma^2(M)$, the new-item variance for a mixed list. Because every slot will be filled with the presentation of some item,

$$\sigma^2(M) = \frac{p}{N}\frac{1-\alpha^{2T}}{1-\alpha^2} + \text{Cov}.$$

The covariance term depends on the spacing of repetitions, which in turn depends on m and R. Define the spacing v where v = T/R so a repeated item occurs every vth position. That is, on average there are v − 1 other items intervening between any two presentations of a repeated item. There are m such repeated items followed (on average) by 0, 1, 2, . . . , m − 1 other items. So, the covariance term is approximately

$$p^2\sum_{l=1}^{m}\alpha^{2(m-l)}\left[\left\{\sum_{r=1}^{R}\alpha^{(R-r)v}\right\}^2 - \sum_{r=1}^{R}\alpha^{2(R-r)v}\right]$$

so the variance is approximately

$$\sigma^2(M) \cong \frac{1}{N}\left[p\frac{1-\alpha^{2T}}{1-\alpha^2} + p^2\frac{1-\alpha^{2m}}{1-\alpha^2}\left\{\left(\frac{1-\alpha^{Rv}}{1-\alpha^{v}}\right)^2 - \frac{1-\alpha^{2Rv}}{1-\alpha^{2v}}\right\}\right].$$

As a check, we ran some computer simulations with a completely randomized presentation order (trial format not preserved), with L = 3; R = 2, 3, 4, or 5; N = 100; α = .95; and p = .5, and the predicted and obtained variances agreed well. Consequently, the derivations may also apply to a randomized presentation order.

Appendix C

In this Appendix, using the continuous memory assumption, we derive explicit expressions for the variances of each of the three experimental conditions of an LSE design as a function of trial block when strength is varied by repetition and averaged over counterbalancing conditions. Assume, as is usually the case, that a within-subjects design is used and that in each block of three trials there is one list of each of the three conditions (weak or W, mixed or M, and strong or S) presented in random (or counterbalanced) order. Let x, y, and z be the variance of each of the three types of lists presented in isolation, where $x = \sigma_W^2$, $y = \sigma_M^2$, and $z = \sigma_S^2$. Define a (column) vector **w** where

$$\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

As shown in Appendix A, at any point in the experiment, the new-item variance is a linear combination of all the possible variance components (i.e., prior items and W, M, and S lists presented to date) weighted by the appropriate interference coefficients, where these interference coefficients reflect the reduction in trace strength due to subsequent list presentations. These interference coefficients will vary with the presentation order and with the six (3!) permutations in each block of three lists. Define vectors **d**, **e**, and **f** for the interference coefficients for the six permutations of the W, M, and S conditions, respectively, where these interference coefficients attenuate u, the variance component from the prior items. As noted in the body of the text,

$$u = \lim_{H\to\infty}\frac{q}{N}\frac{1-\alpha^{2H}}{1-\alpha^2} = \frac{q}{N(1-\alpha^2)}.$$

The three vectors **d**, **e**, and **f** are shown below.

$$\mathbf{d} = \begin{bmatrix} \alpha^{2L} \\ \alpha^{2L} \\ \alpha^{2(T+L)} \\ \alpha^{2(RL+L)} \\ \alpha^{2(T+RL+L)} \\ \alpha^{2(RL+T+L)} \end{bmatrix} \quad \mathbf{e} = \begin{bmatrix} \alpha^{2T} \\ \alpha^{2T} \\ \alpha^{2(L+T)} \\ \alpha^{2(RL+T)} \\ \alpha^{2(L+T+RL)} \\ \alpha^{2(T+L+RL)} \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} \alpha^{2RL} \\ \alpha^{2RL} \\ \alpha^{2(L+RL)} \\ \alpha^{2(T+RL)} \\ \alpha^{2(L+T+RL)} \\ \alpha^{2(T+L+RL)} \end{bmatrix}$$

These three vectors represent the interference coefficients appropriate for a particular condition when that particular condition has been presented as the first, second, or third condition in a particular permutation. When presented first or third, the order of the other two conditions does not matter, but when presented second, the order of the other two conditions does matter.

We also need Matrices A, B, and C for the three conditions (A for W, B for M, and C for S), and these three matrices are shown in Table C-1. For each matrix, the rows are for the six permutations (WMS, WSM, MWS, SWM, MSW, and SMW in that order), and for each row, the columns are the interference coefficients for the three conditions (W, M, and S in that order). If we define a (column) vector **r** as a vector of 1s (**r** is 6 × 1), then the average variance for each of the three conditions in the first block of three trials is

Table C-1
Matrices A, B, and C for the Interference Coefficients for Conditions Weak (W), Mixed (M), and Strong (S), Respectively

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & \alpha^{2L} & 0 \\ 1 & 0 & \alpha^{2L} \\ 1 & \alpha^{2(L+RL)} & \alpha^{2L} \\ 1 & \alpha^{2L} & \alpha^{2(L+T)} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ \alpha^{2T} & 1 & 0 \\ 0 & 1 & \alpha^{2T} \\ \alpha^{2(T+RL)} & 1 & \alpha^{2T} \\ \alpha^{2T} & 1 & \alpha^{2(L+T)} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ \alpha^{2RL} & 0 & 1 \\ 0 & \alpha^{2RL} & 1 \\ \alpha^{2(T+RL)} & \alpha^{2RL} & 1 \\ \alpha^{2RL} & \alpha^{2(L+RL)} & 1 \end{bmatrix}$$

$$\sigma_W^2(1) = \tfrac{1}{6}\mathbf{r}^T(\mathbf{ud} + \mathbf{Aw}) = \tfrac{1}{6}\mathbf{ur}^T\mathbf{d} + \tfrac{1}{6}\mathbf{r}^T\mathbf{Aw},$$

$$\sigma_M^2(1) = \tfrac{1}{6}\mathbf{r}^T(\mathbf{ue} + \mathbf{Bw}) = \tfrac{1}{6}\mathbf{ur}^T\mathbf{e} + \tfrac{1}{6}\mathbf{r}^T\mathbf{Bw},$$

and

$$\sigma_S^2(1) = \tfrac{1}{6}\mathbf{r}^T(\mathbf{uf} + \mathbf{Cw}) = \tfrac{1}{6}\mathbf{ur}^T\mathbf{f} + \tfrac{1}{6}\mathbf{r}^T\mathbf{Cw},$$

where $\mathbf{r}^T$ is the transpose of $\mathbf{r}$ and $T = (R + 1)m$ where m is the number of presentations of the strong items in the mixed-list condition.

For the Bth block of trials, $B > 1$, the vectors ($\mathbf{d}$, $\mathbf{e}$, and $\mathbf{f}$) and the matrices (A, B, and C) have to be modified to take account of the fact that the interference from the prior items is progressively attenuated as B increases and the interference from prior occurrences of the experimental lists is progressively potentiated as B increases. Consequently, one can write

$$\sigma_W^2(B) = \tfrac{1}{6}\mathbf{ur}^T\mathbf{d}(B) + \tfrac{1}{6}\mathbf{r}^T A(B)\mathbf{w},$$

$$\sigma_M^2(B) = \tfrac{1}{6}\mathbf{ur}^T\mathbf{e}(B) + \tfrac{1}{6}\mathbf{r}^T B(B)\mathbf{w},$$

and

$$\sigma_S^2(B) = \tfrac{1}{6}\mathbf{ur}^T\mathbf{f}(B) + \tfrac{1}{6}\mathbf{r}^T C(B)\mathbf{w}.$$

To simplify the notation, let $G = L + T + RL$. Then G reflects the total amount of interference in each block of trials. For the prior items,

$$\mathbf{d}(B) = \alpha^{2G}\mathbf{d}(B - 1), \qquad \mathbf{d}(1) = \mathbf{d},$$

$$\mathbf{e}(B) = \alpha^{2G}\mathbf{e}(B - 1), \qquad \mathbf{e}(1) = \mathbf{e},$$

and

$$\mathbf{f}(B) = \alpha^{2G}\mathbf{f}(B - 1), \qquad \mathbf{f}(1) = \mathbf{f},$$

so the attenuation factor is the same for all three conditions because each block consists of one presentation of each of the three conditions and the presentation order is immaterial. We have then a simple geometric decay in the interference coefficients so

$$\mathbf{d}(B) = \alpha^{2(B-1)G}\mathbf{d},$$

$$\mathbf{e}(B) = \alpha^{2(B-1)G}\mathbf{e},$$

and

$$\mathbf{f}(B) = \alpha^{2(B-1)G}\mathbf{f}.$$

The interference from prior presentations of the experimental lists accumulates over the session, but in a negatively accelerated fashion. In the second block, the variance for each condition is the variance from the second block ($0.167\mathbf{r}^T\mathbf{Aw}$) for the weak condition plus the sum of all three conditions from the first block attenuated (on average) by α^{2G}. In the third block, the variance for each condition is the variance from the third block (still $0.167\mathbf{r}^T\mathbf{Aw}$ for the weak condition) plus the sum of all three conditions from the first block attenuated (on average) by α^{4G} plus the sum of all three conditions from the second block attenuated (on average) by α^{2G}.

More generally, one can write this for the weak condition as

$$\left\{\tfrac{1}{6}\mathbf{r}^T A + \alpha^{2G}\frac{1 - \alpha^{2(B-1)G}}{1 - \alpha^{2G}}(\mathbf{r}^*)^T\right\}\mathbf{w}$$

where $\mathbf{r}^*$ is also a vector of 1s (to sum over the three conditions), but $\mathbf{r}^*$ is 3×1, rather than $\mathbf{r}$ that is 6×1. We have exactly the same expressions for the mixed and strong condition, except we substitute Matrices B or C for Matrix A.

Because the new-item variance for each condition is simply the (weighted) sum of the prior items and the experimental lists, to date we have

$$\sigma_W^2(B) = .167\alpha^{2(B-1)G}\mathbf{ur}^T\mathbf{d} + \left\{.167\mathbf{r}^T A + \alpha^{2G}\frac{1 - \alpha^{2(B-1)G}}{1 - \alpha^{2G}}(\mathbf{r}^*)^T\right\}\mathbf{w},$$

$$\sigma_M^2(B) = .167\alpha^{2(B-1)G}\mathbf{ur}^T\mathbf{e} + \left\{.167\mathbf{r}^T B + \alpha^{2G}\frac{1 - \alpha^{2(B-1)G}}{1 - \alpha^{2G}}(\mathbf{r}^*)^T\right\}\mathbf{w},$$

and

$$\sigma_S^2(B) = .167\alpha^{2(B-1)G}\mathbf{ur}^T\mathbf{f} + \left\{.167\mathbf{r}^T C + \alpha^{2G}\frac{1 - \alpha^{2(B-1)G}}{1 - \alpha^{2G}}(\mathbf{r}^*)^T\right\}\mathbf{w}.$$

This is the variance by the continuous memory assumption for each of the three experimental conditions of an LSE experiment as a function of trial block (B) when strength is varied by repetition. These expressions agreed with several simulations with different values of H (number of prior items), α, N, and L.

With these results, one can easily get an explicit expression for ROR as a function of B because by Equation 1a

$$\mathrm{ROR}(B) = \frac{\sigma_S(B)}{\sigma_W(B)}.$$

These are the values that are entered into Table 2.

It should be noted that the expressions do not just apply to TODAM. Instead, they would apply to any GMM as long as there is a storage equation (to get the interference coefficients) and explicit expressions for x, y, and z, the components of $\mathbf{w}$, for the variance of each of the three experimental conditions presented in isolation. Consequently, it should be relatively easy to find out whether the results of LSE experiments do in fact pose problems for other GMMs.

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